Analysis of M/M/1 Model with Complete Breakdown during Busy Period, Optional Server with Limited Service, Complete Vacation and Delay in Repair

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ABSTRACT

In this paper we have analyzed a single server Markovian queueing model with an optional server for limited-service time, complete breakdown during busy period, complete vacation with some delay in repair. Customers arrival follow Poisson's distribution with rate λ. Service time during busy period is exponentially distributed with rate μ. The server goes under complete breakdown during busy period and hence sent for repairing. During breakdown an optional server with limited-service time is available for serving customers rather than stopping service. This optional server after completing busy period moves to working vacation for limited time period, where service time during this period is slower than busy period service time as server has some prior commitments or work to finish. As the limited-service time finishes, the server goes for a complete vacation and will not serve any customer during this period. If main server gets repaired, it immediately resumes busy period but if some delay occurs in repairing of main server, then optional server (after completing vacation) will act as main server and resumes busy period until main server get repaired. The closed form expression of various system probabilities is derived. Various system performance measures like waiting time, queue length have been evaluated. Finally, some numerical and graphical results have been shown to model the impact of some parameters on different performance measures.

Keywords: *Busy Period; Complete Breakdown; Complete Vacation; Delayed Repair; Limited Service; Optional Server.*

INTRODUCTION

Vacation in queueing theory refers to unavailability of server for service. Complete vacation implies the duration during which server stops serving altogether as oppose to partial or working vacation where server might still serve at reduced capacity. There are many situations when the server is not allowed to operate for more than a specified amount of time in such situation the server need to take a vacation after a specified amount of time. This type of vacation is known as limited-service vacation. The Vacation Queueing model was first introduced by **Levy et al. [18]**. A thorough analysis of the vacation queueing system can be found in the survey paper of **Doshi [15,16]** and researcher's paper of **Tian et al. [12]. Servi et al. [13]** were the first who introduced Working vacation in an M/M/1 queuing model with one and multi working vacation policies. There are many real-life situations where the service gets interrupted due to breakdown in service mechanism. Such type of breakdowns is common in industries, telephone traffic etc. The most common reason of breakdown is hardware malfunction, software crashes or some technical issues. Generally, there are two types of breakdowns: complete breakdown and partial breakdown. When a server breaks down completely, it ceases to serve customers entirely until it is repaired or replaced is called complete breakdown while partial breakdowns refer to a situation when server remains partially operational. Both complete and partial breakdowns introduce uncertainty and variability into queueing system, affecting its efficiency and

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customer experience. When a server goes to a breakdown it needs to be replaced or repaired to restore normal operation. Repair time includes diagnosing the issues, performing the necessary repairs or replacement and ensuring the service is once again operational. This can be seen in case of a machine where some fault may occur in the machine due to continuous working for a long period.

BRIEF LITERATURE REVIEW

Federgruen et al. [14] determined various performance measures in a finite Markovian system with server breakdown during service with repair facilities. **Neuts et al. [17]** analyzed Markovian queue where N servers are subject to breakdown with finite repair facilities. **Yang et al. [10]** investigated a single server markovian queueing model with N policy, working vacation in which breakdown may happens during idle period, working vacation and normal busy period with different rates. **Poonam [6]** analyzed feedback in retrial queueing system with working vacation interruption, Perfect repairing of breakdown server along with the concept of setup time. Further, **Gupta et al. [7]** introduced the concept of waiting server with breakdown and repairing of server in retrial queueing system. **Begum et al. [11]** studied a bulk arrival queueing system with retrial policy and optional service with N policy, uncertainable breakdown and immediate repair. **Melikov et al. [5]** analyzed a multi-server Bernoulli retrial queue with breakdown of non-reliable server during both idle and service time of server. **Bharthidas et al. [9]** analyzed bulk queueing model with Erlangian service with k service phases with system breakdown which leads to repair. **Seenivasan et al. [3]** used Matrix geometric method to derive Probability vector and various system performances in an M/M/1 queue with server vacation, breakdown of server and feedback customer reliant on state of server. **Chakravarthy et al.[8]** discussed a Markovian queue with service time follows phase type distribution, single server taking multiple vacation where a backup server is provided when main server is either goes under breakdown or on vacation. **Hanukov et al. [1]** analyzed a queueing model in which a temporary server with less functionality takes over the primary server during the recovery phase. They used Markov process to analyze the effects of distinct vacation policies in scheduling of period of vacation as a function of the recovery and deterioration rates. **GnanaSekar et al. [2]** analyzed a retrial M/G/1 Queue with Delay in Repair, Feedback with Working Vacation policy along with balking and reneging of customers. **Dasa et al. [4]** derived Transient analysis and various performance measures of an M/M/1/N model with an essential optional service by Runge-Kutta method.

Model Description

In this paper a M/M/1 queueing model with optional server and limited service, complete breakdown during busy period, complete vacation and some delay in repair are considered. The customers arrival follows Poisson distribution with rate λ. Service time follows exponential distribution with rate μ during busy period. Customers receive services on FCFS discipline. During busy period server goes under complete breakdown, therefore needed repair. System breakdown time and repairing time are exponentially distributed with rate α and β respectively. During complete breakdown main server goes to repair, an optional server takes its place and completes busy period. After serving customers in busy period optional server moves to working vacation where rather than stopping service completely to customers, the optional server provided service for limited time period with exponentially distributed rate η, where $\eta<\mu$. If limited-service time provided by optional server completes, then the optional server will go to complete vacation where he will not serve any customer and remains free until the vacation finishes. Vacation time is exponentially distributed with parameter ϕ. During this period if main server gets repaired, it immediately resumes busy period and start serving customers waiting for service but if some delay occurs in repair, then optional server will act as a main server (after completing vacation) and resumes busy period (until main server get repaired) with rate ψ and probability or otherwise it will do some other work assigned to it with probability s=1-r. The transition rate diagram of the model is shown below in figure 1;

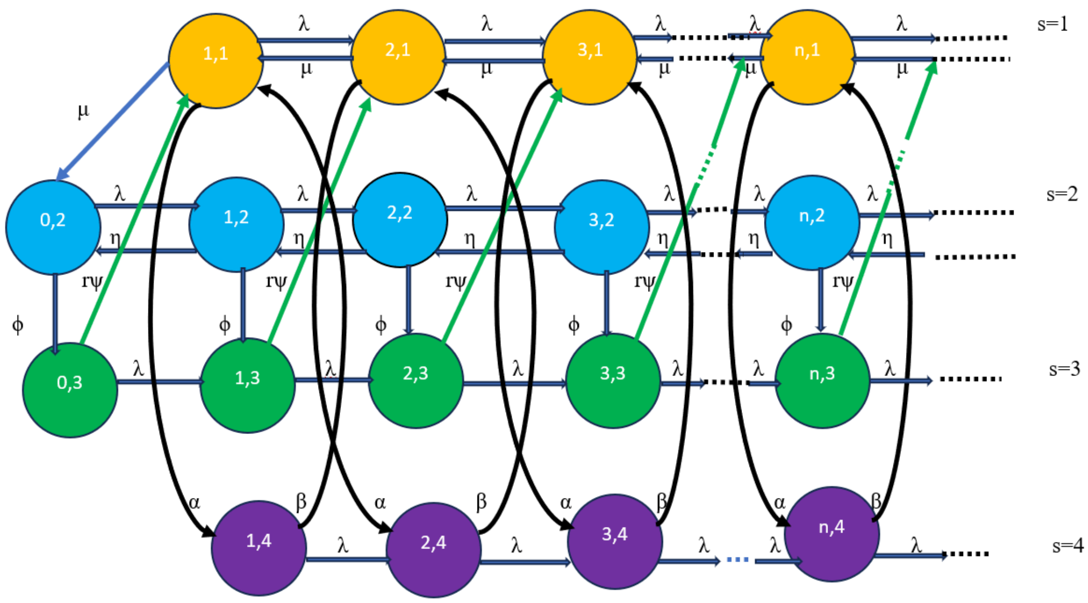

Figure 1: Transition rate diagram of different states of server in the Model.

Mathematical Formulation

Let P_{ns} =P$\{N(t) = n, S(t) = s; n = 0,1,2 \ldots\ldots\ldots; s = 1,2,3,4.\}$denote system's steady state probabilities, where N(t) represents the number of customers in the system at time t, S(t) represents state of server at time t such that

s=1; represents the server is in busy period.

s=2; represents the state when an optional server provides service for limited time period.

s=3; represents the server is in complete vacation.

s=4; represents the server is in repair state.

Now, we develop balance equation for each server state as follows:

For s=1;

$P_{11}(\lambda + \mu + \alpha) = \mu P_{21} + \beta P_{14} + r\psi P_{03}$, n=1 (1)

$P_{n1}(\lambda + \mu + \alpha) = \mu P_{n+1,1} + \lambda P_{n-1,1} + \beta P_{n4} + r\psi P_{n-13}$, n≥2 (2)

For s=2;

$P_{02}(\lambda + \phi) = \mu P_{11} + \eta P_{12}$,n=0 (3)

$P_{n2}(\lambda + \eta + \phi) = \lambda P_{n-12} + \eta P_{n+12}$, n≥1 (4)

For s=3;

$P_{03}(\lambda + r\psi) = \phi P_{02}$, $n = 0$ (5)

$P_{n3}(\lambda + r\psi) = \lambda P_{n-1\,3} + \phi P_{n\,2}$, n≥1 (6)

For s=4;

$P_{14}(\lambda+\beta)=\alpha P_{11}$, n=1 (7)

$P_{n4}(\lambda+\beta)=\lambda P_{n-14}+\alpha P_{n1}$, n≥2 (8)

Some Performance metrics and Steady State Probabilities of System

Now we use PGF method to derive various system Performances and steady state probabilities,

Let $R_1(z)=\sum_{n=1}^{\infty} P_{n1} z^n$, $R_1'(z)=\sum_{n=1}^{\infty} nP_{n1} z^{n-1}$ (9)

$R_2(z)=\sum_{n=0}^{\infty} P_{n2} z^n$, $R_2'(z)=\sum_{n=0}^{\infty} nP_{n2} z^{n-1}$ (10)

$R_3(z)=\sum_{n=0}^{\infty} P_{n3} z^n$, $R_3'(z)=\sum_{n=0}^{\infty} nP_{n3} z^{n-1}$ (11)

$R_4(z)=\sum_{n=1}^{\infty} P_{n4} z^n$, $R_4'(z)=\sum_{n=1}^{\infty} nP_{n4} z^{n-1}$ (12)

Such that $R_1(1)+R_2(1)+R_3(1)+R_4(1)=1$. (13)

Multiply equation (2) by z^n and taking summation over n we get,

$$[\lambda+\mu+\alpha]\sum_{n=2}^{\infty} P_{n1} z^n=\mu\sum_{n=2}^{\infty} P_{n+11} z^n+\lambda\sum_{n=2}^{\infty} P_{n-11} z^n+\beta\sum_{n=2}^{\infty} P_{n4} z^n+r\psi\sum_{n=2}^{\infty} P_{n-13} z^n$$

On simplifying above equation, we get

$$R_1(z)\{\lambda z+\mu z+\alpha z-\lambda z^2-\mu\}=(\lambda+\mu+\alpha)z^2P_{11}-\mu zP_{11}+\beta zR_4(z)-\beta z^2P_{14}$$
$$+r\psi z^2R_3(z)-r\psi z^2P_{03} \quad (14)$$

On further simplifying and taking limit z tends to 1, we get

$$R_1(1)=\frac{1}{\alpha\lambda}[r\psi\alpha(R_3(1)-P_{03})-P_{14}(\lambda^2+\lambda\beta-\alpha\lambda)+2\alpha\lambda P_{11}]=P(B) \quad (15)$$

Where P(B)=Probability of server is in busy period.

Multiply equation (4) by z^n and taking summation over n we get,

$$[\lambda+\eta+\phi]\sum_{n=1}^{\infty} P_{n2} z^n=\lambda\sum_{n=1}^{\infty} P_{n-1\,2} z^n+\eta\sum_{n=1}^{\infty} P_{n+2\,1} z^n$$

On simplifying above equation we get

$$R_2(z)\{\lambda z+\eta z+\phi z-\lambda z^2-\eta\}=(\lambda+\eta+\phi)zP_{02}-\eta P_{02}-\eta zP_{12} \quad (16)$$

Taking limit z tends to 1,we get

$$R_2(1)=\frac{(\lambda+\phi)P_{02}-\eta P_{12}}{\phi}=P(O) \quad (17)$$

Where P(O)=Probability that optional server is providing service for limited time Period.

Multiply equation (6) by z^n and taking summation over n we get,

$$[\lambda+r\psi]\sum_{n=1}^{\infty} P_{n\,3} z^n=\lambda\sum_{n=1}^{\infty} P_{n-1\,3} z^n+\phi\sum_{n=1}^{\infty} P_{n\,2} z^n$$

On simplifying above equation, we get

$$R_3(z)\{\lambda+r\psi-\lambda z\}=(\lambda+r\psi)P_{03}+\phi R_2(z)-\phi P_{02} \quad (18)$$

Taking limit tends to 1, we get

$R_3(1) = \frac{(\lambda+r\psi)P_{03}+\phi\, R_2(1)-\phi P_{02}}{r\psi}$ =P(V) (19)

Where P(V)=Probability that server is availing complete vacation.

Multiply equation (8) by z^n and taking summation over n we get,

$[\lambda+\beta]\sum_{n=2}^{\infty} P_{n\,4}\, z^n = \lambda\sum_{n=2}^{\infty} P_{n-1\,4}\, z^n + \alpha\sum_{n=2}^{\infty} P_{n\,1}\, z^n$

On simplifying above equation, we get

$R_4(z)\{\lambda^2 z + 2\lambda\mu z + \lambda\alpha z - 2\lambda^2 z^2 - \lambda\mu + \beta\lambda z + \mu\beta z - \mu\beta - \lambda\mu z^2 - \alpha\lambda z^2 + \lambda^2 z^3\} = P_{14}\{\lambda^2 z^2 + \lambda\mu z^2 + \alpha\lambda z^2 - \lambda^2 z^3 - \mu\lambda z + \beta\lambda z^2 + \mu\beta z^2 + \alpha\beta z^2 - \beta\lambda z^3 - \mu\beta z - \alpha\beta z^2\} + P_{11}\{\alpha\lambda z^3 - \alpha\lambda z^2 - \alpha\mu z^2 - \alpha^2 z^2 + \alpha\mu z + \alpha\lambda z^2 + \alpha\mu z^2 + \alpha^2 z^2 - \alpha\mu z\} + \alpha r\psi z^2\,\{R_3(z) - P_{03}\}$ (20)

Taking limit tends to 1, we get

$R_4(1) = \frac{1}{\beta\lambda}[\alpha\lambda P_{14} + \alpha\lambda P_{11} + r\psi\alpha\{R_3(1) - P_{03}\}]$=P(R) (21)

Where P(R)=Probability of server is in repair state.

Differentiate equation (16) both side with respect to z and taking limit $z \to 1$, we get,

$R_2'(1) = \frac{1}{\phi}[(\lambda+\eta+\phi)P_{02} - \eta P_{12} - R_2(1)\{\eta+\phi-\lambda\}]$ = E (O) (22)

where $R_2'(1)$ = E (O)=Expected or mean length of queue when optional server is provided service for limited time Period.

Now, differentiate equation (18) once and taking limit z tends to 1, we get

$R_3'(1) = \frac{1}{r\psi}[\lambda R_3(1) + \phi R_2'(1)]$ = E (V) (23)

where $R_3'(1)$ = E (V) =Expected or mean length of queue when optional server is availing complete vacation.

Now, differentiate equation (20) once and limit z tends to 1, we get

$R_4'(1) = \frac{1}{\beta\lambda}[3\alpha\lambda P_{11} + \{\mu\lambda + 2\alpha\lambda - \lambda^2 - \beta\lambda + \mu\beta\}P_{14} + \alpha r\psi\,\{R_3'(1) + 2R_3(1) - 2P_{03}\} - \{2\mu\lambda - \alpha\lambda + \beta\lambda - 2\mu\beta\lambda + \mu\beta\}R_4(1)] =$ E(R) (24)

where $R_4'(1)$ = E(R)=Expected or mean length of queue when main server is in repair state.

Now, differentiate equation (14) once and taking limit z tends to 1, we get

$R_1'(1) = \frac{1}{\alpha}[\beta R_4'(1) - \lambda R_4(1) - (\lambda+\beta)P_{14} + \alpha P_{11}]$=E (B) (25)

where $R_1'(1)$ = E (B)=Expected or mean length of queue in Busy Period.

Now, by using recurrence relation (1), (2), (3), (4), (5), (6), (7), (8) we get

$P_{03} = \frac{\phi}{(\lambda+r\psi)} P_{02} = K_1 P_{02}$, where $K_1 = \frac{\phi}{(\lambda+r\psi)}$

$P_{12} = \left(\frac{\lambda}{\eta}\right) P_{02} = K_2 P_{02}$, where $K_2 = \left(\frac{\lambda}{\eta}\right)$

$P_{11} = \left(\frac{\phi}{\mu}\right) P_{02} = K_3 P_{02}$, where $K_3 = \left(\frac{\phi}{\mu}\right)$

$P_{14} = \frac{\alpha\phi}{\mu(\lambda+\beta)} P_{02} = K_4 P_{02}$, where $K_4 = \frac{\alpha\phi}{\mu(\lambda+\beta)}$

By using above P_{nj}'s, we can rewrite $R_1(1)$, $R_2(1)$, $R_3(1)$, $R_4(1)$ in terms of P_{02} as follows,

$R_2(1) = H_1 P_{02}$, where $H_1 = \frac{(\lambda+\phi-\eta K_2)}{\phi}$

$R_3(1) = H_2 P_{02,}$ where $H_2 = \frac{(\lambda+r\psi)K_1+\phi B_1-\phi}{r\psi}$

$R_4(1) = H_3 P_{02}$, where $H_3 = \frac{1}{\beta\lambda}[\alpha\lambda(K_4+K_3)+r\psi\alpha\{B_2-K_1\}]$

$R_1(1) = H_4 P_{02}$, where $H_4 = \frac{1}{\alpha\lambda}\{r\psi\alpha\{B_2-K_1\}-K_4\{\lambda^2+\lambda\beta-\alpha\lambda\}+2\alpha\lambda K_3\}$

Since $R_1(1)$, $R_2(1)$, $R_3(1)$, $R_4(1)$ and all P_{nj}'s are expressed in terms of P_{02}, therefore we need to calculate P_{02} which can be determined by using Normalization condition,

$R_1(1) + R_2(1) + R_3(1) + R_4(1) = 1,$

$H_4 P_{02} + H_1 P_{02} + H_2 P_{02} + H_3 P_{02} = 1$

$P_{02} = (H_1 + H_2 + H_3 + H_4)^{-1}$

NUMERICAL ANALYSIS

β	P_{11}	P_{12}	P_{03}	P_{14}	P(B)	P(O)	P(V)	P(R)
0.1	0.01	0.24	0.0142857	0.001219	0.0255088	0.3	0.029999	0.127523
0.3	0.01	0.24	0.0142857	0.001162	0.0254484	0.3	0.029999	0.0424140
0.5	0.01	0.24	0.0142857	0.001111	0.0253976	0.3	0.029999	0.0253966
0.7	0.01	0.24	0.0142857	0.001063	0.0253495	0.3	0.029999	0.0181067
0.9	0.01	0.24	0.0142857	0.001020	0.0253096	0.3	0.029999	0.014058

Table 1: Impact of repair rate β on various system probabilities $P_{11}, P_{12}, P_{03}, P_{14}, P(B), P(O), P(V), P(R)$.

It is clear that Table 1 shows impact of repair rate β on various steady state probabilities P_{11}, P_{12}, P_{03}, P_{14} ,P(B), P(O), P(V), P(R) for α=0.5, ψ=0.5,ϕ=0.2,λ=4,η=5,μ=6,$P_{02} = 0.3$,r=0.4.As the value of β increases P_{11}, P_{12}, P_{03}, $P(O)$, $P(V)$ remains constant , a negligible decrement have been noticed in P_{14} ,P(B) while value of P(R) decreases gradually.

α	P_{11}	P_{12}	P_{03}	P_{14}	P(B)	P(O)	P(V)	P(R)
0.2	0.01	0.24	0.0142857	0.000222	0.0295122	0.3	0.299995	0.0098029
0.3	0.01	0.24	0.0142857	0.000333	0.0229569	0.3	0.299995	0.014770
0.5	0.01	0.24	0.0142857	0.000555	0.029845	0.3	0.299995	0.0248402
0.7	0.01	0.24	0.0142857	0.000777	0.030067	0.3	0.299995	0.035087
0.9	0.01	0.24	0.0142857	0.000999	0.0302892	0.3	0.299995	0.0455116

Table 2: Impact of breakdown rate α on different system Probabilities $P_{11}, P_{12}, P_{03}, P_{14}, P(B), P(O), P(V), P(R)$.

It is clear that Table 2 shows impact of Breakdown rate α on various steady state probabilities P_{11}, P_{12}, P_{03}, P_{14}, P(B), P(O), P(V), P(R) for β=0.5,ψ=0.5,ϕ=0.2,λ=4,η=5,μ=6,r=0.4,$P_{02} = 0.3$. As the value of α increases P_{11}, P_{12}, P_{03},P_{14}, P(O), P(V) remains constant, P(R) increases, while minor increment have been noticed in P(B).

α	E(B)	E(O)	E(V)	E(R)	E(L)	W
0.2	0.190502	6	11.9998	0.116638	18.30694	4.576735
0.3	0.193381	6	11.9998	0.1771916	18.37037	4.592593
0.5	0.199183	6	11.9998	0.302899	18.500082	4.625020
0.7	0.265551	6	11.9998	0.4349204	18.700271	4.675067
0.9	0.0943251	6	11.9998	0.5248693	18.618994	4.65474

Table 3: Impact of breakdown rate α on E(B),E(O),E(V),E(R),E(L),W.

It is clear that Table 3 shows impact of Breakdown rate α on system Performances E(B), E(O), E(V), E(R),E(L),W for β=0.5,ψ=0.5,ϕ=0.2,λ=4,η=5,μ=6,r=0.4,$P_{02} = 0.3$.As the value of α increases E(O),E(V) remains constant E(B),E(L) and W increases gradually for α=0.2,0.3,0.5,0.7 and decrement have shown at α=0.9 while E(R) increases continuously.

GRAPHICAL ILLUSTRATION

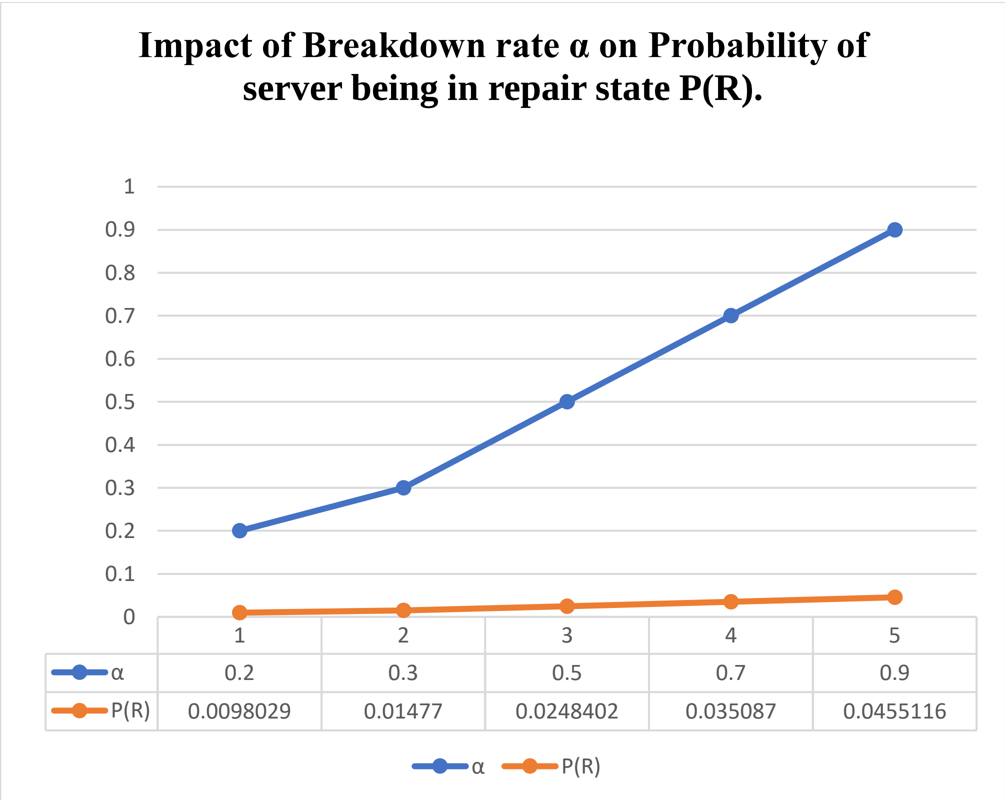

Fig. 2 shows Impact of Breakdown rate α on Probability of server being in repair state P(R).

It is clear that Fig. 2 shows Impact of Breakdown rate α on Probability of server being in repair state P(R) graphically for β=0.5, ψ=0.5, ϕ=0.2, λ=4, η=5, μ=6,$P_{02} = 0.3$,r=0.4. As the value of α increases, P(R) increases gradually.

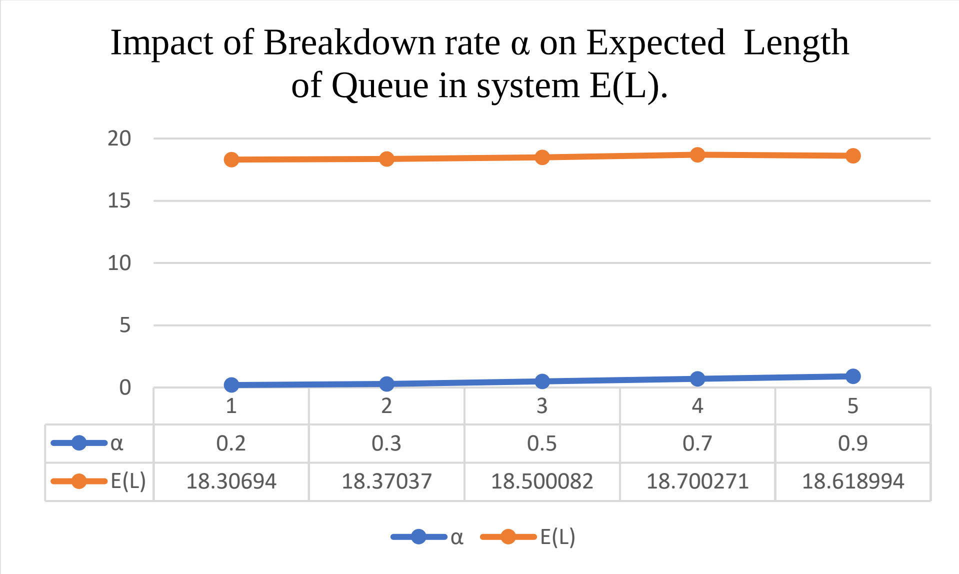

Fig. 3 shows Impact of Breakdown rate α on Expected Length of Queue in the system E(L).

It is clear that Fig.3 shows Impact of Breakdown rate α on Expected Length of Queue in the system E(L) graphically for β=0.5, ψ=0.5,ϕ=0.2,λ=4,η=5,μ=6,$P_{02} = 0.3$,r=0.4.As the value of α increases, minor increment have been noticed in the value of E(L) for α=0.2,0.3,0.5,0.7 and then decrement is noticed at α=0.9.

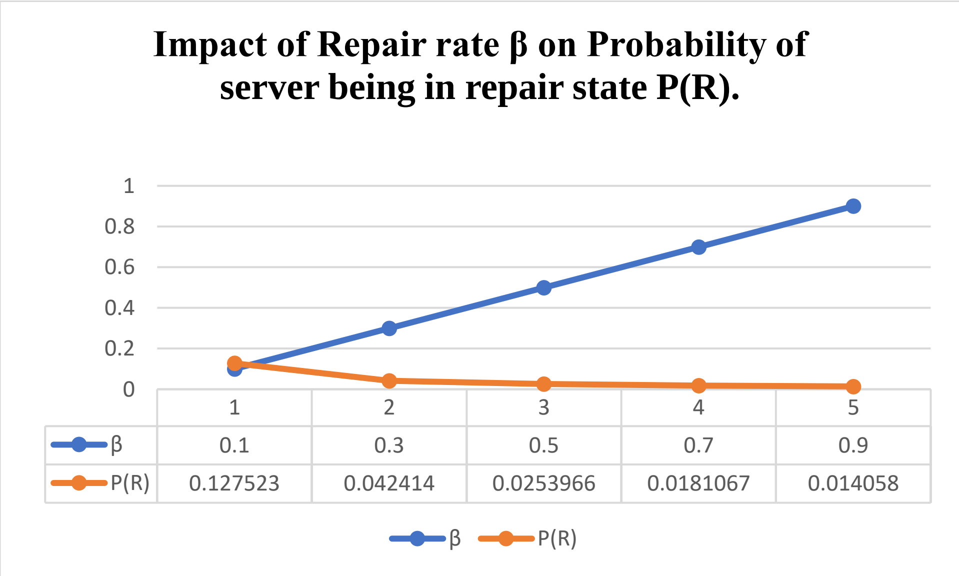

Fig. 4 shows Impact of Repair rate β on Probability of server being in repair state P(R).

It is clear that Fig 4 shows Impact of Repair rate β on Probability of server being in repair state P(R) graphically for α=0.5, ψ=0.5,ϕ=0.2,λ=4,η=5,μ=6,$P_{02} = 0.3$,r=0.4.As the value of β increases, the value of P(R) decreases gradually.

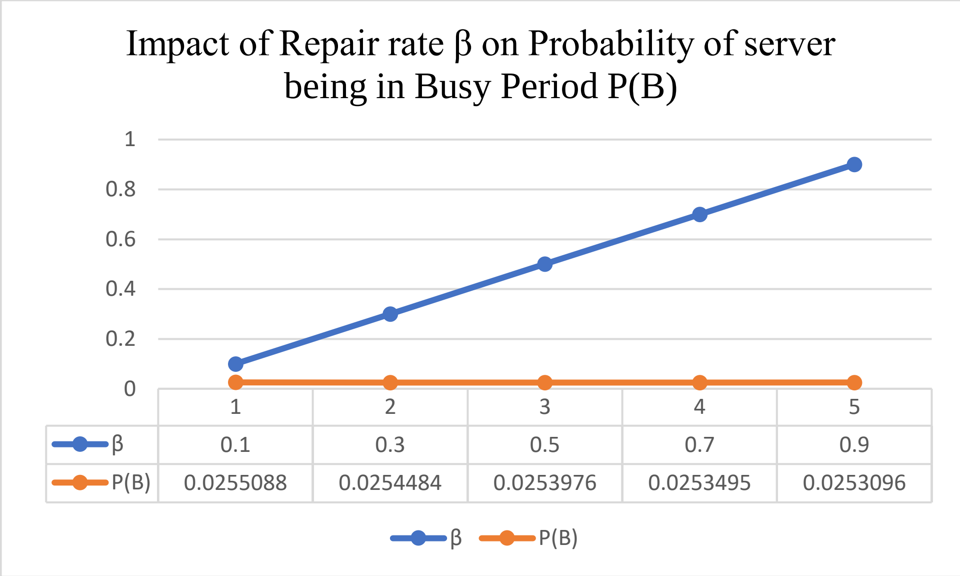

Fig. 5 shows Impact of Repair rate β on Probability of server being in Busy Period P(B).

It is clear that Fig 5 shows Impact of Repair rate β on Probability of server being in Busy Period P(B) graphically for α=0.5, ψ=0.5, ϕ=0.2, λ=4, η=5, μ=6,$P_{02} = 0.3$,r=0.4. As the value of β increases, the value of P(B) almost remains constant.

APPLICATION OF THE MODEL

Consider an emergency department of a hospital where a single nurse provides treatment to the patients on the basis of their condition as they arrive. During high rush hours when main nurse is unavailable due to some reason, an additional nurse may be called upon to assist. The optional nurse has limited availability due to other responsibilities within the hospital. In this scenario, the additional nurse serves as the optional server. However, their availability is limited due to other duties or constraints. This model helps the hospital management to analyze and optimize different aspects of the emergency department operations by evaluating average wait time for patients waiting for treatment, impact of server break down (e.g. a malfunctioning ventilator that lead to longer wait times for patients) and repairing or restoring of server on wait times and patient flow which may help in determination of optimal times or conditions for activating the optional doctor on the basis of patient arrival and workload, impact of the optional doctor/nurse on reduction of wait times and improving patient flow during busy periods. It helps to optimize resource utilization (e.g. Proper allocation of staff and other resources) and enhance patient satisfaction, managing constraints effectively which ensure timely and efficient patient care.

CONCLUSION

In this paper we have analyzed a single server queueing model with complete breakdown during busy period, repair, an optional server with limited-service time, complete vacation with delay in repair. The closed form expression of system probabilities and various system performance measures have been derived by using PGF method. With the help of some numerical results and graphical illustration the impact of some model parameters on different performance measures have been shown. Finally, it has been concluded that breakdown during busy period causes tangible effects on customers, employees and system operation. Therefore, a multifaceted approach, combining technical solutions with strategic management of resources should be use for preventing server from breakdown. An optional or additional server and immediate repair facility should always be available for an unpredictable breakdown as it prevents the system from uncertainty which can affect queueing behavior and performance metrics.

FUTURE SCOPE

This work can be further extended by deriving transient analysis of the model Performances instead of steady state analysis.

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